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CONCEPT OF MATHEMATICAL MODELING IN PREDICTING ENVIRONMENTAL QUALITY

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ABSTRACT

Seeing the increasing pressure of anthropogenic activities on environment it is essential to conserve it from degrading. Many environmentalists are working for proper management of environment. Mathematical models act as a tool in environmental quality prediction as accepted globally. In the present study basic concepts of mathematical modeling for predicting the environmental quality has been discussed. Besides this the application and requirements for developing a model has also been discussed.

INTRODUCTION

Rivers has been used widely for wastewater disposal from industry and municipality throughout India. These disposal increases pollutant loading to the river waters in the recent years due to increase in population, industrial and agricultural activities. Several pollution symptoms including eutrophication, fish kills and lowering of dissolved oxygen below the critical level were observed in the low flow period of the river water. Major Indian rivers flow through areas of large population densities and intensive agricultural activities. The non-point loading of both organic pollutants and nutrients to such rivers throughout its entire stretch is often comparable, if not larger than point loading of pollution from industrial sources or large population centers. Conventional river-cleaning strategies will thus lead to non-attainment of the river cleaning objectives in such cases and, as has been argued, will lead to negative public perception, leading to neglect or abandonment of such schemes due to lack of political and institutional support. Simulation of pollutant movement and its kinetics transformation within the aquatic environment are the key processes in water quality modeling (O'Connor1967). The process has proved as a reliable and economic method of assessing pollutant distribution in surface waters and can be effectively used in management decisions. Although a lot of advancements have been done in this field, little have been carried out in this regard in India.

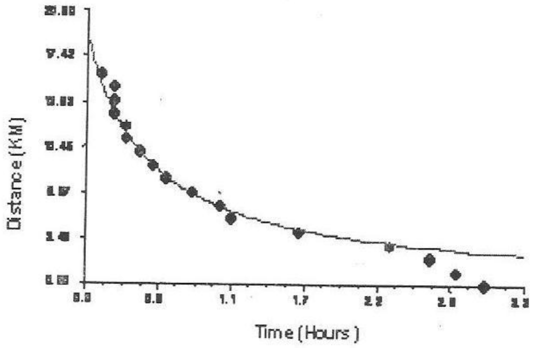
A model is a simplification in which only those components, which are seen to be significant to the problem at hand, are represented in the model. The developments of the mathematical models are a logical development of earlier descriptive tools used to analyse the environment such as drawings, classification and maps. Models should be seen as a complement to other techniques used to arrive at an under standing and they also, we believe uniquely, provide an important means of testing our understanding. Many models are developing to understand the processes of the environment around us. Models can thus be used to evaluate whether the effects and outcomes are reproducible from the current knowledge of the processes. Mathematical models have been developed since the origin of mathematics, but there was a significant increase in modeling activity since the development of calculus by Newton and Leibniz working independently in the second half of the seventeenth century.

MODEL STRUCTURE

Environmental models are focused upon change. This change may be of properties in time or in space. Models are being developed where both temporal and spatial variations are evaluated, so we need to have techniques that can assess both of these changes. The branch of mathematics that allows us to assess change is known as calculus. A basic understanding of calculus is fundamental for all modeling. Model equations are mathematical representations of the processes linking state variables. They are made up of a combination of state variables, parameters and constants, and are formulated to represent, as well as is known (in some cases mechanistically, in other cases empirically) our best knowledge of what controls the rate of processes in the model. In particular, we are careful to ensure that the equations conserve mass. In other words, mass cannot be created or destroyed, but rather moves between the different state variables or across boundaries.

One of the most fundamental concepts in Mathematics, namely the notion of a **Function** is first used by Leibinitz to denote the dependence of one quantity on another. For instance, the dissolved oxygen levels (d_o), which is effected by photosynthesis (p_t) by aquatic plant, Written as d_o p_t or $d_o = k\ p_t$; where k is coefficient for oxygen. Thus we say that d_o is a function of p_t, and to indicate this, we write $d_o = f(p_t)$. The letter *f* does not denote a physical quantity; it just stands for the dependence of one variable quantity on another. In this way we use the basic concepts of developing a mathematical model.

Let us consider another example to understand the concept of mathematical model. We take the velocity of water V_w; which is fined by distance traveled per unit time, which is dynamic in nature. We can write the above relation in differential for $\frac{dx}{dt} = V_w$



The line shows the continuos change in velocity calculated using the formula $\frac{dx}{dt} = V_w$ obtained by differentiation, whereas the points show the measured points at hourly intervals. By the graph we correlate the variable by numerical equation $S = \frac{a}{1 + be^{-ct}}$ This mathematical equation is called as Logistic Model. Where S is Distance, t is time, and a, b and c coefficient.

Most of the models are based on the key principle of the conservation of Mass, energy and momentum. This principle requires that the mass of each water quality constituent being investigated must be accounted for in one way or another. Models can trace each water quality constituent from the point of spatial and temporal input to its final point of export, conserving mass in space and time.Let us consider a simple model for flow, are most often simulated using the St Venant approximations (named after the French hydrologist Jean-Claude Barre de St Venant) and first formulated in 1848. These approximations state that water flows are controlled by the continuity (mass balance) equation

$$\frac{\partial h}{\partial t} + h\frac{\partial u}{\partial x} + u\frac{\partial h}{\partial x} - q = 0$$

$$\underset{(1)}{\frac{1}{g}\frac{\partial u}{\partial t}} + \underset{(2)}{\frac{u}{g}\frac{\partial u}{\partial x}} + \underset{(3)}{\frac{\partial h}{\partial x}} - \underset{(4)}{(S - S_f)} = 0$$

and a conservation of momentum equation made up of the following components:

Where h is depth of flow (m), u is flow velocity (m s^{-1}), t is time (s), x is distance downstream (m), q is unit lateral inflow (m s^{-1}), g is acceleration due to gravity (ms^{-2}), S is the slope of the channel bed (mm^{-1}) and S_f is the slope of the water (mm^{-1}). If we use all of these components (1-4), then the model is known as a dynamic wave approximation (Graf and Altinakar, 1998; Singh, 1996; Dingman, 1984). Component (1) reflects the inertia of the flow; (2) the gravity effect on the flow; (3) the pressure effect; and (4) the friction effect.

APPLICATION OF THE MODEL

The first step in applying the model is analyzing the problem to be solved. What questions are being asked? How can a simulation model be used to address these questions? A water quality model can do three basic tasks describe present water quality conditions, provide generic predictions, and provide site-specific predictions (Ambrose *et.al.*,1988). The first, descriptive task is to extend in some way a limited site-specific database. Because monitoring is expensive, data seldom give the spatial and temporal resolution needed to fully characterize a water body. A simulation model can be used to interpolate between observed data, locating, for example, the dissolved oxygen sag point in a river or the maximum salinity intrusion in an estuary. Of course such a model can be used to guide future monitoring efforts. Descriptive models also can be used to infer the important processes controlling present water quality. This information can be used to guide not only monitoring efforts, but also model development efforts.

Providing generic predictions is a second type of modeling task. Site-specific data may not be needed if the goal is to predict the types of water bodies at risk from a new chemical. A crude set of data may be adequate to screen a list of chemicals for potential risk to a particular water body.

Providing site-specific predictions is the most stringent modeling task. Calibration to a good set of monitoring data is definitely needed to provide credible predictions. Because predictions often attempt to extrapolate beyond the present database, however, the model also must have sufficient process integrity. Examples of this type of application include waste load allocation to protect water quality standards and feasibility analysis for remedial actions, such as tertiary treatment, phosphate bans, or agricultural best-management practices.

Once the network is set up, the model study will proceed through four general steps involving, in some manner, hydrodynamics, mass transport, water quality transformations, and environmental toxicology. The first step addresses the question of where the water goes. This can be answered by a combination of gaging, special studies, and hydrodynamic modeling. Flow data can be interpolated or extrapolated using the principle of continuity. Very simple flow routing models can be used; very complicated multi-dimensional hydrodynamic models can also be used with proper averaging over time and space.

The second step answers the question of where the material in the water is transported. This can be answered by a combination of tracer studies and model calibration. Dye and salinity are often used as tracers. The third step answers the question of how the material in the water and sediment is transformed and what its fate is. This is the main focus of many studies. Answers depend on a combination of laboratory studies, field monitoring, parameter estimation, calibration, and testing. The net result is sometimes called model validation or verification, which are elusive concepts. The success of this step depends on the skill of the user, who must combine specialized knowledge with common sense and skepticism into a methodical process.

The final step answers the question of how this material is likely to affect anything of interest, such as people, fish, or the ecological balance. Often, predicted concentrations are simply compared with water quality criteria adopted to protect the general aquatic community. Care must be taken to insure that the temporal and spatial scales assumed in developing the criteria are compatible with those predicted by the model. Sometimes principles of physical chemistry or pharmacokinetics are used to predict chemical body burdens and resulting biological effects. Computer based models provides this capability by allowing the user to specify the types of distributions and key statistics for any and all input variables. Depending on the specific variable and the amount of available information, any one of several distributions may be most appropriate. A lognormal distribution is the default for environmental and pollutant loadings. In the uncertainty analysis, the distributions for constant loadings are sampled daily, providing day-to-day variation within the limits of the distribution, reflecting the stochastic nature of such loadings.

UNCERTAINTY ANALYSIS

There are numerous sources of uncertainty and variation in natural systems. These include: site characteristics such as water depth, which may vary seasonally and from site to site; environmental loadings such as water flow, temperature, and light, which may have a stochastic component; and critical biotic parameters such as maximum photosynthetic and consumption rates, which vary among experiments and representative organisms. In addition, there are sources of uncertainty and variation with regard to pollutants, including: pollutant loadings from runoff, point sources, and atmospheric deposition, which may vary stochastically from day to day and year to year; physico-chemical characteristics such as coefficients and Henry Law constants that cannot be measured easily; chemo-dynamic parameters such as microbial degradation, photolysis, and hydrolysis rates, which may be subject to both measurement errors and indeterminate environmental controls. Increasingly, environmental analysts and decision makers are requiring probabilistic modeling approaches so that they can consider the implications of uncertainty in the analyses.

CONCLUSION

Hence it is concluded from the above-mentioned techniques, that modeling plays an important role in testing any hypothesis. The approaches of mathematical models may help us to interpret and predict environmental quality and management decisions; besides this it saves time and money. Therefore it is suggested from the study that the gap between mathematics and its application in various fields should be filled by applying mathematical models for proper management of environmental quality.

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